

Chapter 16 — Mixed Proportional Problem Solving — Teacher Guide

Chapter at a glance

Lesson duration	6 lessons, 45–70 minutes each (see pacing table below)
Materials	Classification cards, anchor-model cards from Chapters 1–15, part-percent-whole bars, original-change-final bars, ratio tape strips, unit-rate tables, common-unit comparison tables, proportion tables, coordinate grids, scale-direction strips, zero-origin cards, the two chapter diagrams
Launch question	“A jacket is 20% off, then taxed. Boys:girls = 3:4, total 28. A map uses 1 cm = 15 km. A taxi has a starting fee. What relationship does each one show — before you calculate anything?”
Key representation	The five-step decision routine (name, identify, choose, calculate, check) — keep it visible throughout instruction
Anticipated responses	Correct: classifies the relationship and names the anchor model before calculating, then checks with a second representation. Partial: solves familiar categories correctly but struggles to classify unfamiliar or mixed contexts. Incorrect: applies a familiar procedure immediately without checking whether it fits the structure.
If the readiness check fails	Reteach the specific anchor model named in the routing table (percent, ratio, rate, scale, or proportionality) before returning to mixed practice; do not skip the classification-before-calculation step
Talk prompt	“What relationship do the quantities show, and which anchor model makes that visible?”
Exit ticket	7 items spanning percent, ratio, rate, scale, proportionality, and non-proportionality — each requires naming the model before solving (see Section 14)
Reteach / extend	Reteach: single-category problems with explicit labels, one anchor model at a time.

Extend: problems combining two structures in sequence, and problems where the same numbers admit multiple valid interpretations.
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Purpose and pacing

Chapter 16 is the capstone of the proportional reasoning sequence. It should not be taught as another list of procedures. It should be taught as a decision-making routine in which students classify, model, solve, and check.

Lesson	Focus	Suggested time
Lesson 1	Readiness and classification stations	50–65 minutes
Lesson 2	Percent, ratio, rate, and best-buy mixed problems	55–70 minutes
Lesson 3	Proportions, graphs, equations, and scale mixed problems	55–70 minutes
Lesson 4	Non-proportional relationships and misconception repair	45–60 minutes
Lesson 5	Tripwire, Exit Ticket, and individual routing	50–65 minutes
Lesson 6	Preparation for cumulative review and final mastery assessment	45–60 minutes

Materials

Prepare classification cards, anchor-model cards from Chapters 1–15, part-percent-whole bars, original-change-final bars, ratio tape strips, unit-rate tables, common-unit comparison tables, proportion tables, coordinate grids, scale-direction strips, and zero-origin cards.

Teacher launch guidance

Present the four Tripwire problems without allowing immediate calculation. The important first response is the classification sentence. Ask students to defend why the problem belongs to a category and which representation would make the structure visible.

During mixed work, require students to write a “meaning line” before an equation. For example: “This is a part-to-whole percent problem; 72 is the part and the school population is the whole.” This slows procedural guessing and gives the teacher evidence about the student’s interpretation.

Use paired comparisons between related but non-identical problems. A discount followed by tax changes the base. A ratio of boys to girls is not the same as a ratio of boys to the total. A best-buy problem requires a common unit. A starting fee breaks direct proportionality.

Classification protocol

At the beginning of each problem, students complete the following table:

Prompt	Student response
What quantities are present?	_____
What does each number measure?	_____
What is the whole or base?	_____
What relationship type is present?	_____
Which anchor model fits?	_____
What units should the answer have?	_____
How will I check?	_____

Teacher prompts

Purpose	Prompt
Classify	"What kind of relationship do the quantities describe?"
Choose a model	"Which picture would make the structure visible?"
Identify a base	"What amount is the percent applied to?"
Identify a whole	"What quantities combine to make the total?"
Compare fairly	"What common unit should both options use?"
Track change	"What is the new base after the first step?"
Test proportionality	"What happens at zero input?"
Check scale	"Which direction are you moving?"
Check reasonableness	"Should the answer be larger, smaller, or between known values?"

Differentiation

Students needing concrete support

Use classification stations with only two or three choices at first. Require students to build the anchor model before using symbols. For a mixed problem, students can place the problem card beside a model card and explain the match orally.

Students needing language support

Use sentence frames:

- "This problem is about _____ because _____."
- "The whole/base is _____."
- "The anchor model is _____."
- "I will use _____ because _____."
- "My answer is reasonable because _____."

Students ready to extend

Ask students to design pairs of problems that use similar numbers but require different methods. Ask them to create a non-proportional situation that looks proportional at first, then explain the zero-origin or starting-amount evidence.

Diagnostic rubric

Dimension	4 — Secure and transferable	3 — Secure with routine support	2 — Developing	1 — Not yet secure
Classification	Names the relationship accurately and explains evidence.	Names the correct category with minor prompting.	Confuses related categories.	Begins calculating without classification.
Representation choice	Selects an anchor model that exposes the structure.	Selects a workable model from familiar options.	Selects a model inconsistently.	Cannot connect a model to the context.
Mathematical execution	Calculates accurately and preserves the relationship.	Minor arithmetic errors but sound method.	Method partly correct or incomplete.	Procedure does not fit the problem.
Units and base	Tracks units and changing base throughout.	Usually tracks units and base.	Omits units or uses an earlier base.	Does not identify units or base.
Reasoning explanation	Explains why the method fits and interprets the answer.	Gives a mostly correct explanation.	Explanation is procedural or incomplete.	Cannot justify the solution.
Checking	Uses an appropriate inverse, second representation, or reasonableness check.	Checks when prompted.	Performs a partial check.	Does not check.

Reteaching decisions

If a student scores 1 in classification, return to the station-card sort. If the student scores 1 in representation choice, rebuild one category with concrete materials. If the student scores 1 in units and base, use quantity-and-unit cards and original-change-final bars. If the student scores 1 in checking, require every answer to include a written inverse or zero-origin test.

Revision Support: Expected Ideas and Teacher Moves

Use this compact guide with the new Cumulative Connection and Strategy Studio tasks. Accept equivalent representations when the relationship, units, and reasoning are correct.

Cumulative Connection: Classify a situation as part-whole, ratio, rate, percent, scale, or another relationship before solving. Choose a representation for a missing-value problem and explain why it fits. Estimate the answer to a multi-step proportional problem. Write a final sentence that interprets the answer with units.

Strategy Studio guidance: Expected ideas: $\$160 \times 0.75 \times 1.08 = \129.60 ; without a scale, the map distance cannot be converted to a real distance; equivalent numerical answers must be interpreted with their units; students should classify before selecting an operation. Score the final task for classification, representation, reasoning, computation, and checking.

Transfer Bridge Teacher Guidance

Use the four tasks to distinguish imitation from transfer. Ask students to name the relationship, keep units visible, compare representations, and explain or repair an error. Accept equivalent methods when the structure and interpretation are correct.

Expected guidance: Familiar: classify as multi-step percent; $\$160 \times 0.75 \times 1.08 = \129.60 . Altered: without a map scale, real distance cannot be determined. Non-routine: equivalent numerical answers can describe different quantities and units. Repair: cross multiplication is one tool, not a classification method; first identify the relationship, then select a representation and operation.

Record whether the difficulty is classification, representation, operation, computation, units, interpretation, or checking. Do not assign more routine practice until the source of the error is identified.

Discussion and evidence note

Ask students to name the relationship before calculating, compare at least two representations when appropriate, and revise an explanation after hearing another strategy. Record whether the difficulty is classification, representation, operation, computation, units, or checking.

Answer Key

Ready-to-Learn Check

Percent of a whole.

Ratio with total.

Unit rate or proportion.

Scale.

Multi-step percent.

Proportional, $y = 4x$.

Not proportional.

It makes comparisons fair.

Worksheet A

- 30.** Likely wrong answer: 25% of 120 computed as $120 - 25 = 95$, treating percent as a subtraction. Misconception: not recognizing “percent of” as multiplication by a decimal. Next move: “does 25% of 120 mean subtract 25, or multiply by 0.25?”
- 25%.** Likely wrong answer: 4 (divides 72 by 18 instead of 18 by 72, reversing part and whole). Misconception: dividing in the wrong order. Next move: “which number is the part, and which is the whole?”
- Total parts 5; one part 8; red = 24.** Likely wrong answer: treats “3:2” as if red were of the total, without adding the ratio parts first. Misconception: treating a part-to-part ratio as if the second number were already the whole. Next move: “how many total parts does 3:2 make?”
- \$24.** Likely wrong answer: \$3 (finds the unit rate but forgets to multiply by 8). Misconception: the unit-fraction-only error — stopping before scaling to the requested amount. Next move: “what does one pen cost? Multiply by 8.”
- 4 for \$12 is \$3/item; 6 for \$15 is \$2.50/item, so 6 for \$15.** Likely wrong answer: picks 4 for \$12,

reasoning that \$12 is a lower total price. Misconception: comparing totals instead of unit prices. Next move: “what does one item cost in each deal?”

- 42 km.** Likely wrong answer: 1.17 km (divides instead of multiplying for a map-to-real conversion). Misconception: reversing the scale direction. Next move: “does one centimetre represent a bigger or smaller real distance?”
- 7 cm.** Likely wrong answer: 112 cm (multiplies instead of dividing for a real-to-drawing conversion). Misconception: reversing the scale direction. Next move: “should the drawing be bigger or smaller than the real wall?”
- Yes, $y = 5x$.** Likely wrong answer: “yes” based on the rising pattern alone, without computing $y \div x$ for each pair. Misconception: assuming any increasing table is proportional. Next move: “compute $y \div x$ for every pair — is it the same number?”
- \$48.** Likely wrong answer: \$40 (subtracts 20% as 20 dollars instead of finding 20% of 60). Misconception: treating the percent number as a dollar amount. Next move: “is \$20 the same as 20% of \$60?”
- Total parts are 7, so boys are 2 of 7 parts, or of the class.** Common flaw: states the correct fraction without explaining why is wrong. Next move: ask what the 5 in “2:5” actually represents.

Worksheet B

- 70.** Likely wrong answer: 165 (adds 35 and 200 as if the percent number were a quantity to combine). Misconception: not converting the percent to a decimal before

- multiplying. Next move: "35% as a decimal is what number?"
2. **30%.** Likely wrong answer: 3.33 (divides 150 by 45 instead of 45 by 150, reversing part and whole). Misconception: dividing in the wrong order. Next move: "which number is the part, and which is the whole?"
 3. **80.** Likely wrong answer: 7.2 (multiplies 24 by 0.30 instead of dividing). Misconception: multiplying regardless of which quantity is missing. Next move: "is 24 the part or the whole here?"
 4. **Sale price \$35; tax \$3.50; final \$38.50.** Likely wrong answer: tax calculated on the original \$50 instead of the \$35 sale price. Misconception: not tracking the base across steps — the recurring multi-step percent error. Next move: "what amount does the 10% tax apply to?"
 5. **Total parts 9; one part 7; cats 28 and dogs 35.** Likely wrong answer: treats "4:5" as if cats were of the total, without adding the ratio parts first. Misconception: treating a part-to-part ratio as if the second number were already the whole. Next move: "how many total parts does 4:5 make?"
 6. **3:5.** Likely wrong answer: 9:15 (divides by a common factor but not the greatest one). Misconception: incomplete simplification. Next move: "is there a larger number that still divides both 18 and 30?"
 7. **$8x = 200$, so $x = 25$.** Likely wrong answer: cross-multiplies the wrong pairs, such as $5 \times 8 = 40$, giving a different value. Misconception: incorrect cross-products. Next move: "which two numbers are diagonal from each other?"
 8. **$\$12/h \times 13 = \156 .** Likely wrong answer: \$21 (adds 8 and 13 instead of finding the unit rate and multiplying).
- Misconception: confusing scaling with combining. Next move: "what does the worker earn per hour? Multiply by 13."
9. **$\$12.60 \div 3 = \$4.20/kg$; $\$19.50 \div 5 = \$3.90/kg$, so 5 kg for \$19.50.** Likely wrong answer: picks 3 kg for \$12.60, reasoning that \$12.60 is a lower total price. Misconception: comparing totals instead of unit prices. Next move: "what does one kilogram cost in each package?"
 10. **No; quotients are 6, 4.5, and 4.** Likely wrong answer: "yes," because the output increases by a steady amount (3 each time). Misconception: mistaking a constant difference for proportionality. Next move: "compute $y \div x$ for each pair — do they match?"

Worksheet C

1. **Discount \$180; sale \$720; tax \$86.40; final \$806.40.** Likely wrong answer: tax calculated on the original \$900 instead of the \$720 sale price. Misconception: not tracking the base across steps in a multi-step percent problem. Next move: "what amount does the 12% tax apply to?"
2. **Total parts 12; one part 3; boys 15 and girls 21.** Likely wrong answer: treats "5:7" as if boys were of the total, without adding the ratio parts first. Misconception: treating a part-to-part ratio as if the second number were already the whole. Next move: "how many total parts does 5:7 make?"
3. **cup per muffin $\times 35 = 14$ cups.** Likely wrong answer: 8.75 cups (divides 35 by 4 instead of scaling the unit rate correctly). Misconception: setting up the proportion with the terms reversed. Next move: "does 4 cups correspond to 10 muffins, or does 10 correspond to 4?"
4. **750 mL: \$0. mL; 1.5 L: \$0. mL, so 1.5 L.** Likely wrong answer: compares

- $\$2.85$ and $\$5.40$ directly without converting to a common volume unit. Misconception: comparing incompatible units. Next move: "are both prices measured over the same volume?"
5. **70 km/h $\times$ 9 = 630 km.** Likely wrong answer: 429 km (adds 420 and 9 instead of finding the unit rate and multiplying). Misconception: confusing scaling with combining. Next move: "what is the speed? Multiply by 9 hours."
 6. **Grade 7 = 168; not Grade 7 = 312.** Likely wrong answer: reports 168 as "not Grade 7" instead of Grade 7 itself, reversing which group the percent describes. Misconception: not tracking which part the percent applies to. Next move: "does 35% describe Grade 7, or the rest of the school?"
 7. **800 $\times$ 1.15 = 920; 920 $\times$ 0.90 = 828.** Likely wrong answer: $800 \times 1.05 = 840$ (adds the percentages, 15% – 10% = 5%, instead of applying them successively). Misconception: treating successive percent changes as additive rather than multiplicative. Next move: "does the 10% decrease act on 800, or on the already-increased amount?"
 8. **8.4 $\times$ 15 = 126 km.** Likely wrong answer: 1.79 km (divides instead of multiplying for a map-to-real conversion). Misconception: reversing the scale direction. Next move: "does one centimetre represent a bigger or smaller real distance?"
 9. **Scale factor $42 \div 14 = 3$; height = $21 \times 3 = 63$ cm.** Likely wrong answer: 35 (adds 14 and 21 instead of applying the scale factor). Misconception: confusing scaling with combining. Next move: "what factor changed the width from 14 to 42? Apply that same factor to the height."
 10. **Table: 0: \$6, 1: \$7.50, 2: \$9, 3: \$10.50;** not proportional because output is \$6 at zero. Likely wrong answer: says "yes, proportional" because the table increases by a steady \$1.50 each row. Misconception: mistaking a constant difference for proportionality. Next move: "what is the cost at zero kilometres? Should it be \$0 for a direct proportion?"
 11. **\$121.50. The manager's weekly hours (40) and number of staff supervised (8) were not needed to find the final price.** Likely wrong answer: works the 40 hours or 8 staff into the calculation somehow. Misconception: assuming every number in a word problem must be used. Next move: "what does the question actually ask for? Which numbers answer that?"
 12. **Answers vary; must describe a situation where 240 is a whole amount, 0.30 (30%) is the percent, and 72 is the resulting part, with each meaning stated.** Common flaw: gives a context but doesn't identify all three roles, or confuses the part with the whole. Next move: ask the student to finish three sentences — "240 is __," "30% means __," "72 is __."

Classification and Justification Practice

1. Part-to-whole percent; part-percent-whole bar.
2. Scale; direction strip.
3. Unit rate; rate table.
4. Proportional relationship; table-graph-equation map.
5. Best buy; common-unit table.
6. Ratio with total; tape diagram.
7. Multi-step percent; original-change-final model.

8. Non-proportional starting amount; zero-origin test.

Tripwire and Exit Ticket

Tripwire: Problem 1 is multi-step percent and gives \$81.36; Problem 2 is ratio with total and gives 12 boys; Problem 3 is scale and gives 67.5 km; Problem 4 is non-

proportional because the output is \$4.50 at zero input.

Exit Ticket: 27 and percent model; total parts 7, one part 7, boys 14 and ratio-with-total model; 63 km/h and 504 km; 78 km; yes, $y = 4.5x$; starting fee means the output is nonzero at zero input; name quantities, classify, choose an anchor model, calculate, attach units, and check.